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ABSTRACT

Reported is an attempt to examine the nature of the reasoning used by college students who consistently erred on timer tape problems and a comparison of their performance with that of students who made logical responses. The author suggests that the poor performance of students, classified as intuitive responders, is a result of their use of an incorrect solution strategy rather than as a result of cognitive immaturity or inability to mentally coordinate information about two dimensions. (Author/SA)

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Intuitive and Logical Reasoning Approaches to Multi-Dimensional Problems

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Intuitive and Logical Reasoning Approaches to Multi-Dimensional Problems

Abstract

College students in non-science classes answered questions concerning the printed record a described piece of physics equipment would produce for an object whose speed varied in a specified manner. The problems did not require a knowledge of physics but did require thinking in terms of a system of three variables and predicting the impact of a change in one of these variables on the system. Forty percent of the subjects gave systematically incorrect responses corresponding to an hypothesized intuitive bias toward focusing on the positive pole of a physical dimension and associating the positive poles of different dimensions with each other.

The present study examines the reasoning of young adults on problems requiring the coordination of two changing dimensions. As a matter of convenience, these will be referred to as "dimensional coordination" problems. When given a new type of dimensional coordination problem, a person must represent the variables involved in working memory and at the same time keep in mind the relationship among them, transform the first variable in accord with the problem specification, and then figure out the new status of the other variables. Much of the content of a course in economics, mathematics, or physics consists of dimensional coordination problems, and students are often expected to be able to mentally figure out the qualitative impact of a change in one variable on the other variables in the system.

The dimensional coordination problems used in the present study involve interpreting timer tapes made by a piece of physics lab equipment used for studying motion. A paper tape is attached to one end of a moving object (e.g., a weight sliding down an inclined plane). The tape passes through a timing mechanism that produces a dot on the tape at constant time intervals. In the laboratory, students interpret the pattern of dots on the tape to see how the object's speed is changing over time. In the paper and pencil problems, students are presented with hypothetical situations, such as a weight moving down an inclined plane, and instructed to select from a group of timer tapes the one that could have been produced in that situation.[1] Fuller, Karplus, and Lawson (1977) found that many high school and college students perform poorly on this type of problem, even after a course in physics, and attributed such failures to the fact that many students were not reasoning at the level of Piagetian formal operations.

The research reported here is an attempt to examine more closely the nature of the reasoning used by students who consistently err on timer tape

problems and to compare their performance with that of students who make logical responses. Given that many students are not answering these problems correctly, it is important to locate the source of their difficulty.

One factor to be considered is the demand that such problems place on working memory. Research on human information processing has demonstrated that there are limits to the number of pieces of information a person can keep in mind and "work on" at one time (Baddeley & Hitch, 1974; Miller, 1956; Norman, 1976). This capacity limitation appears to take a toll in complex reasoning tasks where the difficulty of solving a problem is often directly related to memory load (Denny, 1969; Simon & Kotovsky, 1963). Figure 1 contains a hypothetical model of how people might coordinate information to solve timer tape problems. In the model, steps 5 and 6 require the coordination of information about several variables operating in a system. Whether or not the model is correct in its particulars, any logical approach to solving timer tape problems would seem to require keeping several factors in memory at one time in order to coordinate changes in their states.

Pilot testing with the timer tape task suggested that something more than memory load is involved in unsuccessful performance, however. Students were not just making frequent errors; different students were all making the same kind of mistake and making it over and over. The tapes they selected were the qualitative opposites of the correct tapes, i.e., where they should have chosen a tape with dots getting progressively denser they chose one with dots getting progressively further apart and vice versa. Such consistency suggested that the timer tape problems were triggering an intuitive, inadequate strategy in these students. The present study was undertaken to verify the existence of this ineffective strategy and to shed light on its nature.

Considerable research describes the characteristics of intuitive strategies in children, but less effort has been directed toward describing the pre-existing strategies adults bring to various types of problems. A tendency that seems to appear in the thinking of adults as well as children is an inclination to group together separate dimensions, treating them as one on the assumption that more of one will always go with more of the other. Examples in children's thinking can be pointed to in the familiar Piagetian tasks (e.g., the preoperational child's tendency to equate the number of discrete items arranged in a linear array with the length of the array, and to treat the volume of some substance within a container as equivalent to the height of the substance within the container). In an area closely related to the present investigation, Siegler and Richards (1979) found that when asked to observe two electric trains moving along parallel tracks, young children judged the train that stopped at a point farther along on its track as having gone faster, farther, and for a longer time, regardless of its starting point or speed. Even for the older subjects in the Siegler and Richards study, the hardest problems were those in which the two dimensions that had to be coordinated did not vary together and had to be considered separately (e.g., where the train that moved for a longer time did not also travel a longer distance).

This kind of thinking is observed in children when dealing with content for which adults have clearly differentiated concepts (e.g., number and length of an array), but adults appear to exhibit similar thinking on other types of problems. For instance, when asked to judge whether two events or traits are correlated with each other, most adults do not compare the probabilities for joint and separate occurrence (Smedslund, 1963).

According to Shweder (1977) people instead rely on their beliefs about the extent to which the two events or traits resemble each other, and conclude that like events will co-occur. Information about the joint occurrence of the two things receives attention; the proportion of cases in which one thing occurred without the other gets little regard. In the area of person perception, for example, this tendency leads to the assumption that such traits as friendliness and helpfulness will always go together. In this way, the perceiver can deal with fewer personality dimensions in judging people, reducing the number of separate variables that must be kept track of. Research on this tendency to treat different personality dimensions equivalently indicates that the strength of this tendency differs markedly among different raters and is stable enough to be treated as an individual difference (Kelly, 1970).

To take an example from another area in human reasoning, in concept formation tasks where the concept to be learned involves several attributes (e.g., "red" and "square"), people look for conjunctive concepts, like red and square, before considering disjunctive ones, like red or square (Hunt & Hovland, 1960). A conjunctive concept is a kind of lumping together (red square) that lessens the number of separate pieces of information the subject has to keep in mind in evaluating feedback from the experimenter. Looking for conjunctive concepts makes the task easier and is facilitative when the experimenter's concept is in fact a conjunctive one, but produces inefficient performance when the concept is disjunctive.

If applied to timer tape problems, this type of thinking would entail directly associating the dimension being measured--speed of the moving object--with the appearance of the dots on the timer tape. If subjects assume that more of one dimension--speed--always goes with more of another--

dots--they can answer timer tape questions without ever coordinating multiple dimensions. Figure 2 illustrates what this intuitive algorithm looks like. The model is simpler than the logical algorithm presented in Figure 1, and like the grouping tendencies in person perception and concept formation discussed above, should pose less of a burden for working memory. The problem, of course, is that the answers generated with the intuitive model are always incorrect except in the case of an object moving at a constant speed. If some students are using this hypothesized intuitive strategy, their responses to timer tape problems should be not just poor in general, but inaccurate in predictable ways. For a gradually increasing speed, for example, subjects using this intuitive strategy would select a tape with dots becoming gradually denser rather than gradually more sparse. For any other pattern of changes in speed, the tapes that would be selected using an intuitive and a correct strategy can be similarly predicted.

Method

Subjects

Forty-three students in undergraduate educational psychology classes at a public university served as subjects. All but six of the subjects were female.

Materials and Procedure

A series of nine timer tape problems was devised. Each problem consisted of a description of some moving object, e.g., "a cart rolling faster and faster down an inclined plane." As in the example, the change in the speed of the object was always made explicit so that students did not need to know anything about physics or the laws of motion in order to answer correctly. Students were provided with illustrations of five timer tapes from which to select one appropriate to the described pattern of motion. One tape contained dots spaced at constant intervals; one had dots getting progressively closer together; one had dots getting progressively farther apart; one had dots at a fixed short interval for half the tape and at a fixed longer interval for the other half; the fifth tape contained this last pattern in reverse. After choosing a timer tape appropriate to the described situation, students were asked to explain their selection. Figure 3 contains a set of five timer tapes and a sample question.

Problems were presented in a test booklet containing one constant rate problem and two problems corresponding to each of the other four timer tapes, one problem to a page. The first two pages of the booklet contained instructions, a description of the timer mechanism's operation, and a self-test on the workings of the timer with feedback provided. The self-test required students to indicate that as the object moved the paper (not the dot marker) moved with it and that dots were made on the paper tape at constant time intervals. The self-test questions might be expected to draw subjects' attention to the need to consider the specific nature of the timer mechanism and reduce their inclination to make rapid intuitive responses to the timer tape problems. However, the questions were considered necessary in order to be sure that the students understood the

way the timer mechanism worked, amd thus that systematically incorrect answers could be attributed to an alternative solution strategy rather than just to confusion about the nature of the mechanism. Two forms of the booklets were used with two different random orders for the timer tape illustrations and the nine problems.

Results and Discussion

Tape Selection Responses

Subjects encountered little difficulty in understanding the operation of the timer mechanism. The initial responses to the self-test questions were correct for all but three students (who were given the correct answers and instructed to reread the description of the device). Nevertheless, errors on the timer tape problems themselves were frequent. These errors were classified as qualitative, quantitative, or both. Qualitative errors entailed selecting tapes such that close-together dots corresponded to high speed. Quantitative errors involved selecting a tape where the pattern of quantitative changes in spacing was incorrect (e.g., selecting a tape depicting a switch between two constant rates rather than a gradual change in speed). On each of the eight problems involving a change in speed, one of the five tapes was the correct response (tape D for the problem in Figure 3); one was a qualitative error (tape A); two were qualitatively correct but quantitatively incorrect (tapes B and E); and one tape was a double error in that it was incorrect both qualitatively and quantitatively (tape C in Figure 3). Selecting the constant rate tape when a variation in speed was specified (e.g., selecting tape E for the problem in Figure 3) was designated a type II quantitative error while the selection of a tape showing a gradual shift in speed rather than a sudden one or vice versa (e.g., selecting tape B in Figure 3) was termed a type I quantitative error.

Type II quantitative errors always involved selecting the same constant rate tape (for a given version of the test booklet) while for the other error types, the corresponding tape varied from problem to problem (since the described pattern of motion varied). Students using either an intuitive or a logical reasoning strategy should answer the constant rate problem correctly; and in fact, all 43 subjects did choose the correct tape for this problem. The constant rate problem was not included in any of the data analyses. The two forms of the test booklet did not differ in terms of the number of errors they elicited, $t < 1$, or in the number of qualitative errors, $t(42) = 1.11$, and responses on the two forms were combined for all other analyses.

Table 1 presents the proportion of responses of each type students made for the eight problems involving a change in speed. Errors of some kind were made on 59 percent of these problems. Computation of the 95 percent confidence interval for the proportion of errors that were of each type indicated that qualitative errors were the most common ($.39 < p < .56$). Double and quantitative type I errors were about equally common ($.23 < p < .35$ and $.15 < p < .25$, respectively) while quantitative type II errors were more rare ($.02 < p < .08$). Of the errors committed, 75 percent involved a qualitative confusion such that increases in speed and dot density were positively associated (qualitative plus double errors).

The tape selection responses were examined also at the subject level. The assumption that different students are using different strategies to answer these questions (the logical or the intuitive model as illustrated in Figures 1 and 2, respectively) becomes untenable if students are not responding consistently according to one or the other. An individual subject should make qualitative errors on virtually all the changing speed

problems or on few or none of them. Students were classified as logical if six or more of their eight responses were qualitatively correct (correct or quantitative error responses). Students were classified as intuitive responders if six or more of their responses were qualitatively incorrect (qualitative or double errors). (The probability that a random responder would fit the criterion for classification as an intuitive responder is approximately $.04)^2$ In line with the assumption that students are using one of the two types of models to work these problems, individual subjects were generally quite consistent in their responses; 39 of the 43 students matched the criterion for one of the two classifications. Twenty-two students (51 percent) were classified as logical while 17 (40 percent) were classified as intuitive responders. The proportion of intuitive responders was reduced somewhat from the level found in pilot testing where problems were presented in a more casual setting without the elaborate explanation and self-test on the workings of the timer mechanism but was still sizable. The observed proportion of intuitive responders far exceeded the chance level of 4 percent, $Z = 11.97$, $p < .0001$.

Verbal Explanations

Students were classified as logical or intuitive reasoners a second time on the basis of their verbal explanations for their tape selections. These classifications were made without reference to the type of tape selection response made. Verbal references to increases in speed producing dots farther and farther apart were classified as logical and statements linking speed increases with "more dots" or dots closer together were classified as intuitive. Two sample logical explanations are: "the faster the object the more the tape will be pulled through before the timer

makes another dot on the tape" and "the dots start close then sequentially get farther apart indicating an increase in speed." Sample intuitive explanations are: "the dots are close for high speed and then distant for low speed" and "the dots keep getting closer together towards the end, indicating faster movement."

Each subject was classified on the basis of those problems for which he or she had given an independently interpretable verbal explanation.[3] A few subjects gave intuitive explanations on some problems and logical explanations on others. These subjects and those who gave no independently classifiable verbal explanations were not classified. Under this procedure, 32 of the 43 students could be classified on the basis of their verbal explanations: 18 were logical and 14 were intuitive in their explanations.

A comparison of the subject classifications based on verbal explanations with those made on the basis of tape selection responses showed that 31 students were classifiable by both criteria and in all 31 of these cases the two classifications matched. Only one student who could not be classified on the basis of tape selections could be classified on the basis of her explanations and that student gave intuitive explanations. Eight students with no classifiable verbal explanations had been classified on the basis of their tape selections: four were logical responders and four were intuitive. Three students could not be classified by either set of criteria.

Most of the classifiable explanations, whether intuitive or logical, referred to the density or "closeness" of the dots on the timer tape. There were other specific references in many explanations, however, which did differ between intuitive and logical problem solvers. The hypothetical model for solving timer tape problems by coordinating dimensions depicted in Figure 1 contains steps involving consideration of the speed of the

paper tape as it moves through the timer mechanism. The intuitive model contains no reference to the movement of the tape at all. Although a request to explain one's answer is unlikely to elicit a report of the student's complete thinking process, it seems reasonable to expect that the verbal explanations of logical responders should be more likely to mention the movement of the paper tape than would the explanations of intuitive responders. Going through the verbal explanations to look for such explicit references confirmed this expectation. Ten students mentioned the movement of the tape in one or more explanations and all of these subjects had been classified as logical on the basis of their tape selections.

On the other side of the coin, intuitive responders should be more likely to show evidence in their explanations of directly equating the dots with the movement of the object without considering the movement of the tape or the action of the marking mechanism as intervening variables. This turns out to be the case: Five students referred to the dots themselves as being "fast" or "slow." An example of this kind of response is "In the beginning the dots are very close because they're moving fast." Four of the students who made this kind of reference were intuitive responders while the fifth could not be classified on the basis of tape selections. None of the students who referred to the dots as having speed made logical tape selections.

These students showed a tendency also to describe the dots as "looking" fast or slow. These explanations were reminiscent of the kinds of reliance on physical resemblances Shweder (1977) found in judgments of co-occurrence likelihood. For some reason, "lots of dots" is apparently what fast movement "looks like" to some students.

> Dots are quick then being spaced at regular intervals looks like train slowed down

> Marble rolls faster with inclination E [tape E] looks like it indicates this--dots get closer together and are more
>
> Because the dots in the beginning look like a brisk jog & then they separate for the slow walk

In order to estimate the pervasiveness of this feeling that closely spaced dots look fast, a separate sample of 27 undergraduates in an education class were shown two timer tapes, one with dots spaced at equal long intervals and one with dots spaced at equal short intervals. The students were asked to label each drawing as "fast dots" or "slow dots", whichever seemed more appropriate. They were not told what the drawings represented or anything about the timer tape mechanism. Students were surprisingly uniform in their choice of labels for the two drawings. Twenty-four of the 27 students labelled the widely spaced dots as "slow" and the densely spaced dots as "fast." Hence, students apparently have a pre-existing inclination to equate closely spaced marks with high speed, an inclination that would work against them when given timer tape problems to solve.

While the hypothesized intuitive strategy accounted for a large number of errors in timer tape selections, there were also many responses that were quantitatively incorrect. The students' verbal explanations clarified some of these. There was a tendency to add something to the problem as stated and then to select a timer tape on the basis of this added information. For example, given the problem "marble picking up speed as it rolls freely down an inclined chute", one student explained her tape selection as follows: "The marble has a constant speed until it rolls down." With her addition to the problem, she had a different pattern of speed changes to find a tape for, and it's not surprising that she committed a quantitative error.

The poor performance of students classified as intuitive responders on these problems is viewed here as a result of their use of an incorrect

solution strategy rather than as a result of cognitive immaturity or inability to mentally coordinate information about two dimensions. Once the correct strategy was explained to intuitive responders, they seemed to understand it and to realize that their own responses had been incorrect. Their problem appeared to be not that they couldn't apply a logical step-by-step strategy to solve these problems but rather that they did not even try to do so; the problems evoked intuitive responses and these subjects felt no need to think about them further. Intuitive responders typically report that the problems are very easy to figure out and appear quite confident about their responses until the correct solution strategy is explained. These findings suggest that many students, even at the college age, are inclined to approach certain problems at a shallow level, positing correspondences between positive poles of various dimensions as a means of simplifying problems, allowing solution through a simple direct translation of the dimension specified as changing with the dimension of interest. Directions for further investigation include looking at the range of problems on which this type of thinking is prevalent, and testing whether the tendency is more widespread on problems with salient visual features (like the timer tapes), whether this type of intuitive responding is a stable individual difference, and whether the tendency can be modified through appropriate training.

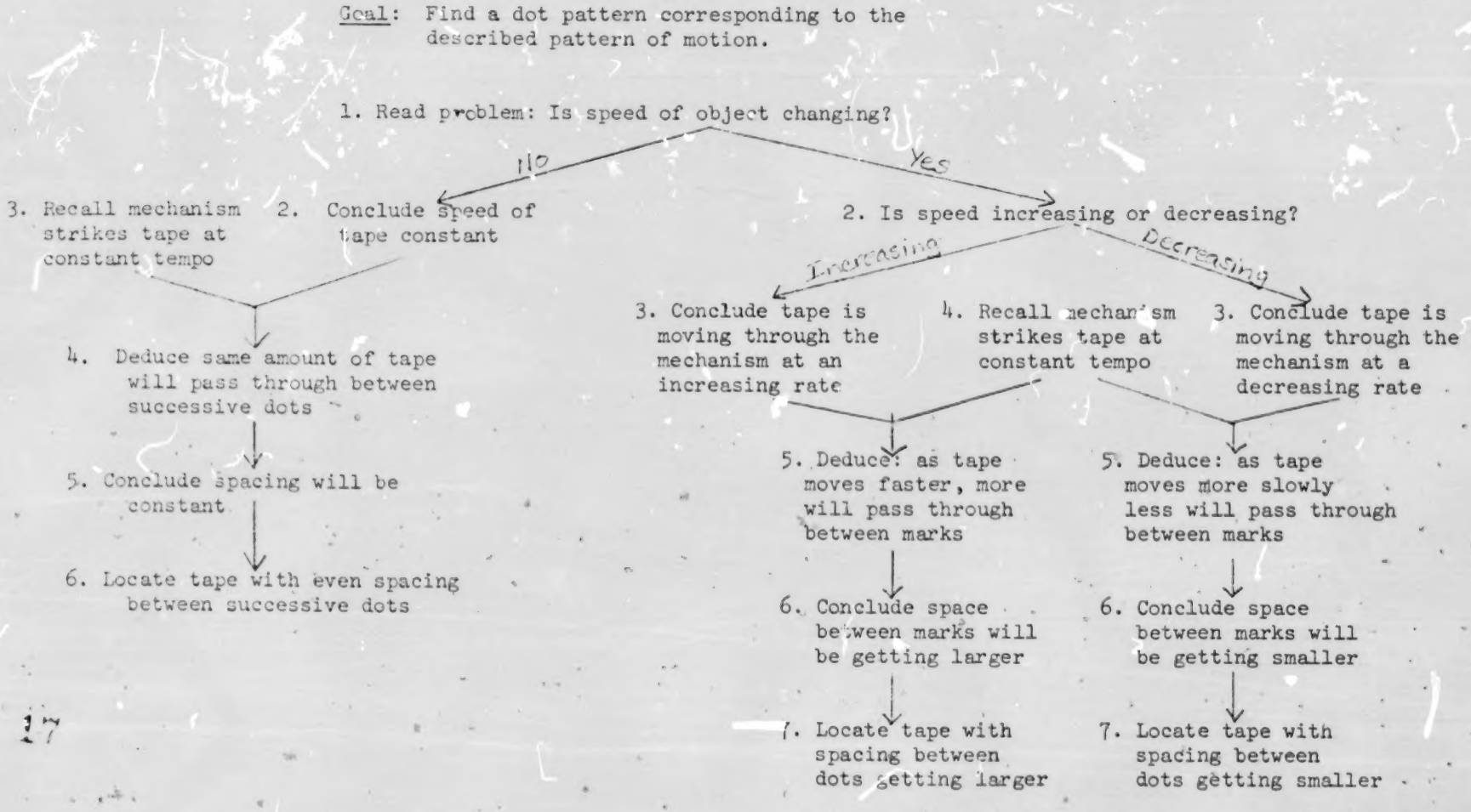

Figure 1. Model for solving timer tape problems by coordinating multiple dimensions.

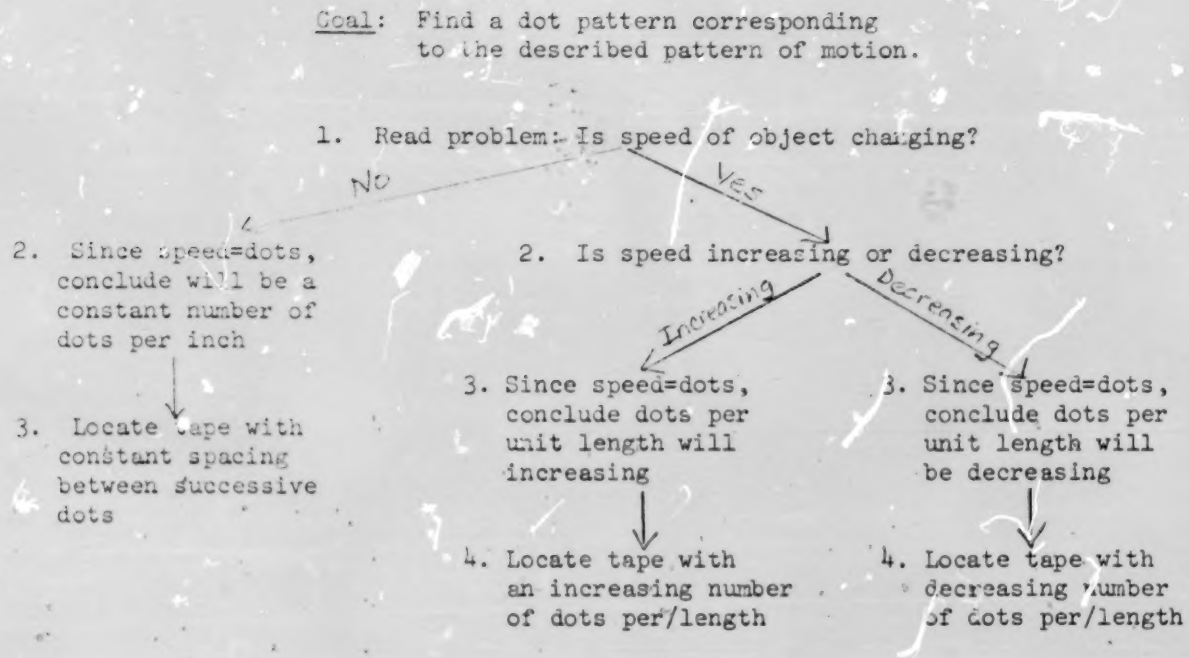

Figure 2. Intuitive (incorrect) model for solving timer tape problems.

TIMER TAPES

Circle the letter indicating the timer tape that fits the situation described below.

An electric train gradually losing speed as it moves up a steep hill.

A B C D E

Explain why the timer tape you selected fits this example:

Figure 3. Timer tape illustrations and sample problem.

Table 1

Responses to Timer Tape Problems, by Type

Response Type	Proportion of Responses
Correct	.41
Quantitative Error (Type I)	.12
Quantitative Error (Type II)	.03
Qualitative Error	.26
Double Error	.16
No Answer	.02

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Footnotes

[1] These problems were adapted from the "ticker tape puzzle" used in the original version of the Workshop on Physics Teaching and the Development of Reasoning, American Association of Physics Teachers, Anaheim, California, 1975.

[2] The multinomial probability that a random responder would make a qualitative or double error on six or more of the problems was computed with the formula:

$$\binom{8}{6}(.40^6)(.60^2) + \binom{8}{7}(.40^7)(.60^1) + \binom{8}{8}(.40^8)(.60^0) = .042141.$$

[3] Many of the verbal explanations could not be classified without depending on knowledge of the subject's tape selection for clarification. In these explanations the relationship the student was assuming between changes in speed and changes in dot pattern was not explicit. Many of these were in effect restatements of the problem (e.g., "dots indicate a gradual decline in speed"). Others referred only to the quantitative aspects of the student's tape selection (e.g., "slows down from friction; is gradual"). These explanations were not classified since they could have been categorized only by referring to the tape selection the student made, thus repeating the response classification already carried out.